Critical Review: Modeling of Acceleration Effects on Solid Propellant Combustion

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Acceleration fields normal and into burning solid propellant surfaces have been observed to cause marked increases, often time-dependent, in their regression rates. Models have been developed for prediction of this burning rate augmentation as a function of various parameters for metalized propellants and non-metalized composite propellants. Most of the metalized propellant models are invalid in that they assume a steady-state involving a finite number of metal agglomerates of constant size controlling the burning rate by formation of pits which lead the burning: in reality, such a steady-state cannot be achieved and flooding of the propellant surface with metal occurs. One model does present a reasonable approach to treating the transient behavior in a semiquantitative fashion, although the problem of calculating agglomerate size as a function of time and various independent parameters is not resolved. Two models of the interaction for nonmetalized propellants appear to be physically unrealistic.

Nomenclature

A_i = area of heat transfer from particle to propellant in Glick model (Fig. 1)
A_s = projected area of cone base (Fig. 1)
a_p = major radius of distorted sphere (Fig. 2)
B = group defined by Eq.(16)
C_d = drag coefficient
d_{fv} = fuel bubble initial diameter
D_g = diffusivity of fuel vapor in oxidizer vapor
D_{pit} = diameter of pit where it intersects propellant surface
d_{oc} = coarse oxidizer diameter
e = eccentricity of distorted sphere
f = fraction of metal burned at surface of propellant (equal to 0 at zero acceleration)
f_H = fraction of energy release from oxidizer particle burning near surface which is transferred back to the propellant
G = fraction of particles emerging from cone surface which are added to agglomerate
G_a = fraction of particles emerging anywhere in pit which are added to agglomerate
G_r = Grashof number, $\alpha\rho_{v,ox}\Delta\rho d^3_{fv}/\mu_g^2$
h_c = energy release associated with oxidizer particle burning near surface per unit mass consumed
h_v = heat to raise unit mass of propellant to surface temperature and gasify it
Δh = enthalpy driving force from agglomerate in pit to propellant surface
ΔH_c = heat of combustion (per unit mass) of particle (agglomerate)
J = fraction of fine oxidizer reacting near surface
k_b = constant in d^2-law for aluminum burning, $d_0^2-d^2 = k_b t$
L = heat transferred to surface per unit mass of metal burned there
m = defined by $f_H = kP^{(1/m)}$
$\dot{m}_c$ = mass burning rate of particle (agglomerate) in a pit
m_i = initial agglomerate weight
m_p = agglomerate weight
n = burning-rate pressure exponent
n_0 = burning-rate pressure exponent in absence of acceleration field
N_i = number of initial agglomerates per unit area of propellant
N_s = number of pits per unit area of propellant surface
P_c = freestream pressure
$P(\eta)$ = local pressure in gap between agglomerate and pit base
Pr = Prandtl number
$\dot{Q}_{MP}$ = excess enthalpy transmitted back to propellant from particle in a pit
$\dot{Q}_{PF}$ = heat transfer rate from gas-zone flames to propellant surface
$\dot{r}$ = propellant linear burning rate with acceleration field
$\dot{r}_o$ = propellant linear burning rate in absence of acceleration
$\dot{r}_a$ = linear burning rate at pit bottom in direction of the acceleration field
r_p = particle radius
r_{pc} = critical particle radius (larger ones retained, smaller ones leave surface)
r_{pm} = mass median particle (or agglomerate) radius
Re_o = a Reynolds number, $\rho_s \dot{r}_o d_{fv}/\mu_g$ used in Eqs. (28) and (29)
Sc = Schmidt number
Sh = Sherwood number
T_{flame} = final flame temperature
T_{surf} = propellant surface temperature
U_{gs} = gas velocity away from propellant surface
V_{wedge} = volume of wedge of propellant shown in Fig. 1.
V_{fv} = velocity of fuel bubbles away from surface
W = weight fraction metal in propellant
W_{oc} = weight fraction of coarse oxidizer in propellant
W_{of} = weight fraction of fine oxidizer in propellant
ΔX_{flame} = average distance of gas-zone flames from propellant surface
α = acceleration magnitude

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δ_r = distance from surface to point of total consumption of fuel bubbles (Glick model)
$\delta(\eta)$ = agglomerate thickness at η
η = distance (from center) along gap between particle and pit base (Fig. 2)
η_H = $W_{of}h_c/h_v$
η_0 = η for which pit wall slope becomes constant at $\cos^{-1}(\dot{r}_o/\dot{r})$ (Fig. 2)
λ = gap width between particle and pit base (Fig. 2)
λ_g = thermal conductivity of propellant flame gases
μ_g = gas viscosity
ρ_g = gas density
ρ_o = oxidizer particle density
ρ_p = metal particle or agglomerate density
ρ_s = propellant density
$\rho_{v,ox}$ = oxidizer vapor density
$\Delta\rho$ = fuel vapor density minus oxidizer vapor density
σ = logarithm of geometrical standard deviation of particle size (log-normal distribution)
σ_s = surface tension of liquid agglomerate
τ_{fv} = lifetime of fuel bubble
ϕ = angle between acceleration vector and normal to propellant surface (Fig. 1)

Introduction

ACCELERATION forces directed into a propellant burning surface have been observed to generally result in an increase in burning rate. The magnitude of the change in burning rate has been found to depend on acceleration level, the base (no acceleration field) propellant burning rate, pressure, initial temperature, and propellant composition parameters such as oxidizer particle size, binder type, and additive (e.g., aluminum) loading. The interaction of acceleration fields with solid propellant burning rate is quite important in ballistic tailoring of missile systems, particularly for spin-stabilized missiles employing internally-perforated grains. If burning rate augmentation due to acceleration fields is not properly allowed for by the designer, there may result thrust-time traces outside acceptable bounds for the given mission or, in extreme cases, motor overpressure to the point of bursting the motor case. Thus, it is important that the motor designer have tools (models) to enable him to predict the response of candidate propellants to acceleration effects. In addition, such models can help the propellant formulator tailor formulations to minimize their sensitivity to a given acceleration environment. Major conclusions which appear to be supported by a majority of the experimental studies of this phenomenon[1] are: 1) Both metalized and non-metalized propellants exhibit sensitivity to acceleration fields directed normally into their burning surfaces, burning rate increasing with acceleration magnitude; 2) Metalized propellants are generally more sensitive to acceleration than non-metalized propellants, particularly at low g-levels; 3) The mechanism by which the burning rates of metalized propellants are altered involves retention of metal and metal oxide at the surface, agglomeration of the metal, and formation of pits below the agglomerates caused by increased local heat transfer back to the decomposing propellant surface; 4) The acceleration sensitivity of metalized propellants is time dependent; 5) The effect of pressure level on acceleration sensitivity varies from propellant to propellant; 6) Increased base burning rate leads to decreased acceleration sensitivity.

Several models have been developed in an attempt to describe the phenomenon of burning rate alteration by gravitational fields for metalized propellants[2-12], and two models have been developed for non-metalized composite propellants[9,10,13-15]. In all of the metalized propellant models, the effect of acceleration on the burning rate through mechanisms other than ones directly involving the metal is neglected or ignored—this may be a serious shortcoming in that there are probably metal-dependent and metal-independent mechanisms operating simultaneously.

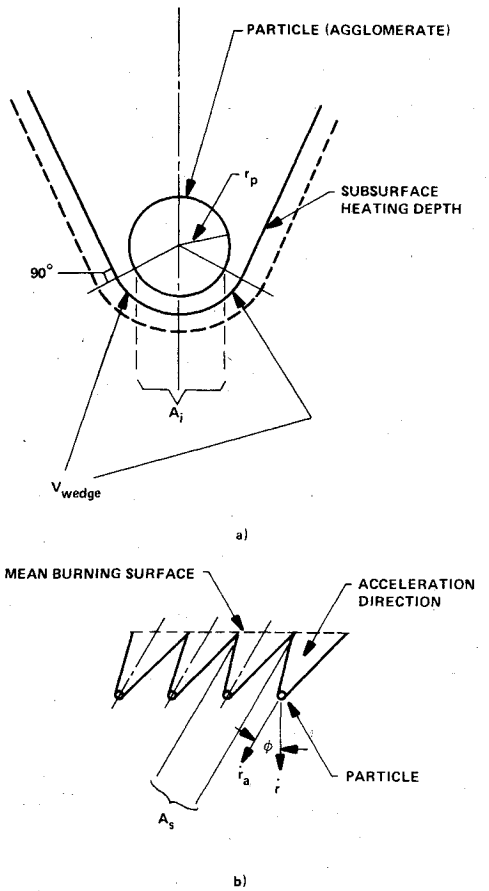

Fig. 1 Analytical model of Glick, et al.,[11] for acceleration effects on metalized propellant combustion.

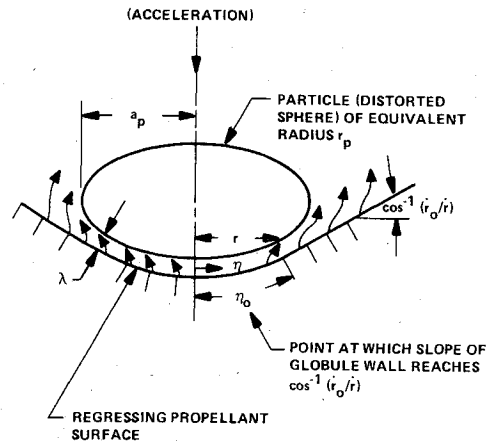

Fig. 2 Proposed physical model—Crowe and Willoughby's second model.[3]

Metalized Propellant Models

The first model for acceleration-induced burning rate alteration in metalized propellants was developed by Crowe and Willoughby[2] in 1966. This model is based on the assumption that metal particles and/or agglomerates of greater than a critical size are prevented from leaving the propellant surface by an acceleration field directed into the surface until they burn down to a size where the body force resulting from the g-field is equaled in magnitude by the drag force resulting from combustion gases flowing away from the surface. The resulting extra surface combustion of aluminum is postulated to increase burning rate by increasing surface heat release (this extra heat is assumed to be distributed uniformly, a poor assumption in light of experimental data discussed later), thus

increasing the rate of heatup and gasification of the solid propellant ingredients. The heat balance is written by Crowe and Willoughby[2] as

$$\rho_s(\dot{r}-\dot{r}_o)h_v=\rho_s\dot{r}WLf \tag{1}$$

This equation is simply rearranged to yield a burning-rate augmentation ratio of

$$\dot{r}/\dot{r}_o=1/[1-(WL/h_v)f] \tag{2a}$$

Writing the surface heat balance in the form of Eq. (1a) involves an implicit assumption that heat transfer from the gas flame to the surface ($\dot{Q}_{PF}$) is fixed at a given pressure independent of the change in burning rate accompanying an acceleration field, a poor assumption. In reality, heat balances should be written as

$$\rho_s\dot{r}h_v=\rho_s\dot{r}WLf+\dot{Q}_{Pf} \tag{3a}$$

$$\rho_s\dot{r}_o h_v=\dot{Q}_{PF,o} \tag{3b}$$

where

$$\dot{Q}_{PF}=k(T_{\text{flame}}-T_{\text{surf}})/\Delta X_{\text{flame}} \tag{4}$$

But, it is readily shown that at a given pressure, the "average" flame offset distance, ΔX_{flame}, is proportional to the burning rate, $\dot{r}$, as long as the mechanism of the gas-phase burning is not altered (a basic assumption of the model). Thus, it may be shown, using Eqs. (3a, 3b, and 4) that Eq. (2a) should be modified to

$$\dot{r}/\dot{r}_o=1/[1-(WL/h_v)f]^{1/2} \tag{2b}$$

That is, the additivity of the basic heat flux from the "normal" propellant flame and the flux from the metal burning on the surface has been incorrectly treated in this model.

The quantity "f" is dependent on the average metal particle (or agglomerate) size, the size distribution, and a quantity denoted the "critical particle size." The critical particle size is the size which will result in the drag force on the particle due to gas flow away from the surface being precisely off-set by the body force resulting from a g-field directed into the propellant surface. Particles smaller than the critical size will leave the surface. Details of the derivation of an expression for "f" as a function of metal particle or "agglomerate" size, pressure, propellant burning rate, and acceleration level are reviewed in Ref. 1, where it is pointed out that since the model provides no information regarding the dependency of agglomerate size on the various independent parameters, it cannot really be said to shed much light on the dependency of augmentation ratio on these parameters.

Another shortcoming of this model is that it considers the excess heat release from metal particle combustion at the surface to occur uniformly, rather than at discrete locations (pits) on the surface as first observed by Northam[16] and later by other investigators in examination of quenched propellant surfaces. Accordingly, Glick and co-workers[9-11] developed a second model for metalized propellants in 1966. The basic assumptions involved in development of this model are: a) Condensed phase particles large enough to be retained at the surface (in pits) agglomerate into one large particle in each pit; b) The interaction between the retained condensed phase and the burning surface occurs at only a finite number of points; c) The large single agglomerate in each pit leads to increased regression rate through combustion and consequent extra heat input to the surface at the bottom of the pit; d) The line of descent of the agglomerates into the propellant is colinear with the g-field. (Fig. 1); e) The process is steady in the mean: i.e., the rate of addition of fresh metal to the agglomerate in each pit equals the agglomerate combustion rate; f) Retention of a fresh metal particle exposed at the wall of the pit and subsequent incorporation into the agglomerate is based on the relative magnitude of g-forces and gas flow viscous forces on the particle.

A detailed critical review of this model is also presented in Ref. 1. The final burning rate augmentation expression derived is

$$\dot{r}/\dot{r}_o=(\dot{r}_a/\dot{r}_o)\cos\phi=\cos\phi\left[\frac{1-W\Delta H_c(1-G)/2h_v}{1-W\Delta H_c[1+((\cos\phi/N_s\pi r_p^2)-1G]/2h_v}\right]^{0.5} \tag{5}$$

where:

$$G=0.5\,\text{erfc}\left\{\frac{1}{\sigma}\,ln\,\frac{(9\mu_g\dot{r}_o\rho_s/2\rho_p\rho_g\alpha)^{1/2}}{r_{pm,o}}\right\} \tag{6}$$

(The subscript zero for the particle radius denotes virgin particles from the propellant as opposed to agglomerate particles.) This author, in deriving Eq. (6), finds $\sigma\sqrt{2}$ in place of σ in the denominator of the error function.

As may be seen, there are two quantities in Eq. 5 which are not known a priori, namely N_s and r_p – since they appear as a product term, they may be replaced by one quantity for data fitting purposes. However, one gains from this analysis no insight as to how this lumped parameter might vary with acceleration magnitude or direction, pressure, propellant composition, initial particle size, oxidizer particle size, etc. Thus, the results of this analysis are not particularly useful, simply replacing our ignorance of how the augmentation rate varies with various parameters with ignorance of how the agglomerate size-pit density product varies with these parameters.

Experimental observations by Willoughby, Crowe and Baker[3] and Anderson and Reichenbach[17] that inert additives such as Al_2O_3 and tungsten result in the same type of burning-rate-acceleration interaction as aluminum negated models based on metal combustion at the propellant surface and led to a third model by Crowe and Willoughby[3-5] in 1968 in which the agglomerate is assumed to lead to increased burning rate by serving as a thermal short circuit between the region of complete combustion and the comparatively cold propellant surface. It is assumed that the agglomerate is buoyed above the bottom of the pit by the propellant gases being generated below it (see Fig. 2) and that its temperature is essentially equal to the overall propellant flame temperature. Since the agglomerate will be liquid at this temperature, the acceleration forces will tend to cause it to assume the shape of an oblate spheroid as shown in Fig. 2. In order for the gases generated at the propellant surface to flow out through the narrow gap between the agglomerate and the base of the pit (slot width λ) the pressure in the slot must rise toward the center. Thus, the pressure-area integral over the entire agglomerate leads to a net upward force which is equated to the downward directed acceleration body force. In employing the Navier-Stokes equations for cylindrical flow to calculate the pressure distribution beneath the distorted sphere and subsequently performing the force balance, Crowe and Willoughby make several simplifying assumptions whose validity is somewhat questionable:

1) The gap width is independent of radial position between $\eta=0$ and $\eta=\eta_0$ (Fig. 2). Actually, since the particle is fluid and can therefore assume a nonelliptical shape, and since the burning surface can adjust its contour, it is not a priori obvious that the slot width should be constant—instead, it probably widens from the center out to the edges to partially accommodate the increasing mass flow through it. The gap width distribution should actually be determined by a technique of minimization of total energy (potential energy

associated with distortion of the globule from spherical shape, potential energy associated with displacement of c.m. in a centrifugal field, potential energy associated with the pressure field in the gap, and kinetic energy of gases in the gap) with variational calculus procedures being used to determine the $\eta(r)$ function which minimizes the energy.

2) The gas density in the gap is assumed to be independent of η. Actually, it is probably better to assume that the gas temperature and molecular weight are independent of η and thus that gas density is proportional to pressure.

3) The gas-efflux velocity from the propellant surface is independent of η. Actually, the burning rate measured normal to the surface decreases with increasing η (shifting from $\dot{r}$ at $\eta=0$ to $\dot{r}_o$ at $\eta=\eta_o$ as may be easily shown through geometrical arguments), tending to decrease the gas efflux velocity with increasing η.

Before the Navier-Stokes equations can be solved, the eccentricity of the particle (agglomerate) must be determined. This is done by application of a potential energy minimization (minimization of potential energy associated with surface tension forces plus that associated with displacement of c.m. in a centrifugal field, but neglecting the potential energy associated with the pressure field) technique yielding a relationship between the particle's major radius and the radius of an equivalent sphere:

$$a_p/r_p = f_1(3\sigma_s/\rho_p r_p^2 \alpha) \tag{7}$$

Use of the Navier-Stokes equations and a pressure-area integral vs body force balance then yields a relationship between the gap width, the propellant burning rate under the acceleration field and various independent parameters which may be combined with Eq. (7) to eliminate (a_p/r_p), yielding

$$\lambda = f_2(\dot{r}, r_p, P_c, \alpha, \text{propellant constants}) \tag{8}$$

An energy balance is written to provide another relationship between λ and $\dot{r}$. This balance relates the increased heat feedback requirements of the propellant below the agglomerate (to raise $\dot{r}_o$ to $\dot{r}$) to the increased heat transfer rate from the flame zone above the propellant through the agglomerate and across the narrow gap of width λ to the propellant surface. (The same analysis of flow through the gap that was used to calculate the pressure distribution on the underwide of the agglomerate is used to calculate the thermal resistance across the gap). The following form of equation results:

$$\rho_s(\dot{r}-\dot{r}_o)h_v = f_3(\lambda, \text{constants}) \tag{9}$$

Again, in Eq. (9), we are faced with the same dubious assumption as in the previous models, namely the assumption that all other heat feedbacks are unchanged (at constant pressure) by the change in burning rate.

The two equations relating the burning rate and the gap width are combined by Crowe and Willoughby to yield

$$(2/9)^{1/4}(r_p/a_p)(\rho_g\rho_p\alpha/r_p)^{1/4}\,\mu_g^{1/2}K/\rho_s\dot{r}_o(1-W) = A(\dot{r}_o/\dot{r}, e) \tag{10}$$

where r_p/a_p is determined from Eq. (7) and the eccentricity, e, is simply related to r_p/a_p by geometrical considerations. The function A is plotted against $\dot{r}/\dot{r}_o$ for several values of e in Fig. 3. For $\dot{r}/\dot{r}_o > 1.2$,

$$A \approx 0.2 + 0.5[(\dot{r}/\dot{r}_o) - 1] \tag{11}$$

leading to (with the $(2/9)^{1/4}$ lumped into the empirical constant, K)

$$\dot{r}/\dot{r}_o = 2[(r_p/a_p)(\rho_p\rho_g\alpha/r_p)^{1/4}(\mu_g^{1/2}K/\dot{r}_o\rho_s(1-W))] + 0.6 \tag{12}$$

Specification of the propellant (composition, particle sizes, etc.), operating pressure, and $\dot{r}_o$ thus permits calculation of burning rate augmentation vs acceleration magnitude, if r_p is known. However, the agglomerate size may be, and in fact quite likely is, a function of acceleration, pressure, base burning rate, etc. Without characterization of its dependency on such variables as acceleration magnitude, pressure, et al., Eq. (12) can therefore not be used meaningfully to predict burning rate augmentation as a function of these variables.

In an attempt to resolve this problem, King and McHale[6,7] pursued the following development in 1969 to relate the agglomerate radius to the various independent variables. First, it was assumed that the agglomerate size in each pit eventually reaches a steady-state value at which point the rate of metal addition to the agglomerate is equal to the rate of mass removal by combustion. (A buoyancy force-body force balance on each fresh metal droplet appearing on the wall of the pit was used to determine whether or not the droplet is added to the agglomerate.) It was further assumed that the oxide product droplets formed near the upper half of the agglomerate are sufficiently small that they are able to leave the pit along with the gaseous propellant products. Aluminum combustion on the bottom half of particle (in the narrow slot area) was assumed to be negligible, the rationale for this assumption being that aluminum oxide initially formed in this region would be unable to escape and would quickly form a protective coating on the agglomerate. Based on aluminum particle combustion studies by Macek[18], a d^2-law was used to describe the aluminum combustion. With these assumptions, for G approximately equal to one, the number of pits per unit area of propellant and the steady-state agglomerate size are related by:

$$\dot{r}\rho_s W = (\Pi/4)N_s\rho_p k_b r_p \tag{13}$$

where k_b is the empirical burning-law constant in the d^2-burning law $(d_o^2 - d^2 = k_b t)$, obtainable independently from Macek's data. Eqs. (12 and 13) were combined (for $\dot{r}/\dot{r}_o > 1.2$) to yield:

$$(\dot{r}/\dot{r}_o)^{5/4} - 0.6(\dot{r}/\dot{r}_o)^{1/4} = 2\left[\frac{r_p}{a_p}\right]\left[\frac{\rho_p^2\rho_g\alpha\Pi k_b N_s}{4\rho_s W}\right]^{1/4}\frac{\mu_g^{1/2}K}{\rho_s\dot{r}_o^{5/4}(1-W)} \tag{14}$$

Next, a procedure for calculation of N_s, the number of pits per unit area was developed for completion of the analysis. The approach taken (described in Ref. 1, 6 and 7) involved use of Crump's "pocket model"[19] for calculation of an average agglomerate size on a surface without pits (initial condition) and a force balance to determine how many agglomerates are removed from that surface and how many remain to initiate pits.

When this model was developed and programed, it was found that the particle burning rate was not sufficient to permit a steady-state agglomerate size to be reached for acceleration levels in excess of 5 to 10g. To remove dependency of the analysis on the technique used to calculate the pit density, agglomerate size data obtained by Northam[16] from examination of residues were examined to determine whether particles in the measured size range could reach a steady-state size with consumption rate limited by the d^2-law with Macek's constants—the answer was again negative. In fact, it was found that burning would have to be allowed over the entire surface (top and bottom) at a rate approximately ten times that predicted from d^2-law extrapolation of Macek's data to permit steady-state sizes in the range of the measured residue sizes to be obtained. Thus, the analysis indicated that the agglomerate size and, hence, the burning rate augmentation were likely to be time-dependent and that any model based on a steady-state agglomerat size was doomed to failure.

Confirming this conclusion, photographic, interrupted-burning, and residue examination procedures used by investigators at UTC[3-5], NASA/Langley[20-23], and the U.S. Naval Postgraduate School[13,17,24,25] show that the burning rate behavior of metalized propellants in acceleration fields is indeed time-dependent due to continuous change in the size and nature of the metal agglomerates on the propellant surface. In general, burning rate increases early in the burning to a maximum and then decreases, finally leveling off at a value above the nonacceleration rate.

The results of a photographic study of metalized strand combustion in a g-field are particularly interesting[3-5]. "The films show that aluminum agglomerates which also formed on the burning surface under static conditions but were quickly entrained by the combustion gases, were held on the surface and coalesced into burning globules when acceleration forces were present. The retention of these globules resulted in the formation of pits in the propellant surface because of the increase in burning rate near the globule. The globules increased in size by collecting aluminum agglomerates emerging from the pit walls. The pits continued to grow due to increased heat transfer to the propellant surface beneath the burning globules. When the pits covered a large part of the surface, they began to merge, forming still larger globules which flattened out in the acceleration field. As combustion proceeded, the pits continued to merge and the globules became flatter until they covered the surface." These observations logically explain the time-dependent behavior of burning rate augmentation of metalized propellants noted previously by the USNPGS investigators. The UTC investigators distinguish five stages of combustion and agglomeration: 1) Initial agglomeration; 2) Pit formation; 3) Flattening and growth of pits; 4) Coalescence of pits; 5) Total surface flooding.

It is noted that the time required to reach a given stage is dependent on propellant formation, pressure level, and acceleration level. This is particularly important in that agglomerate size is an important parameter in the equations for burning-rate augmentation, and the above information indicates that it is a strong function not only of time, but of many other variables.

The interrupted-burning work by Northam[20-23] is also of interest. As the time allowed for burning before interrpution is increased, the nature of the surface of the extinguished propellant and the metal/metal oxide residue on it is found to change, indicating a change in mechanism with increased time. For very short burn times, the residue is in the form of spheres; as time increases these become buttons, large irregular pieces, and finally sheets. In addition, with increase in time, the pits found in the extinguished surface broaden and flatten out and finally disappear. This behavior is entirely consistent with the observations of the UTC investigators in their photographic study. Also, the residue weight increases with increasing g-loading and, more important, there is a shift in the nature of the residue from spheres to buttons to sheets. Thus it appears that a model in which agglomerate size is considered to be independent of g-loading cannot possibly succeed. Any successful analysis of the augmentation process for metalized propellants must involve an unsteady-state analysis of considerable complexity.

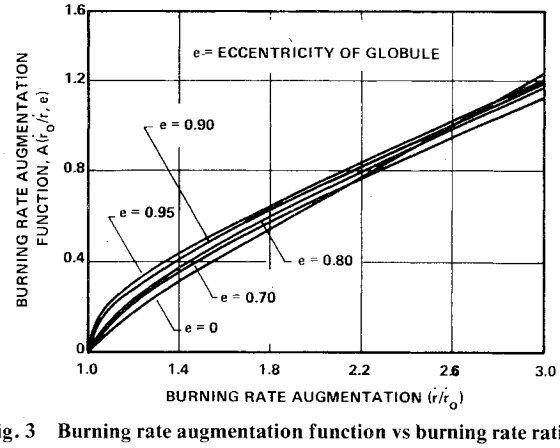

Fig. 3 Burning rate augmentation function vs burning rate ratio.[3]

In 1972, Crowe[12] published a "unified" model of acceleration-induced burning rate augmentation of metalized propellants capable of predicting time-dependent behavior. The bulk of this analysis is quite similar to that in the previously described Crowe and Willoughby second model.[3-5] Crowe does not attempt to model the initial rise in the burning rate vs time curve, but begins his modeling at the burning rate peak, using at this point the previous Crowe and Willoughby analysis, except for the assumption that the agglomerates are still spheres rather than oblate spheriods. As a result he obtains a slightly modified form of Eq. (10)

$$\dot{r}/\dot{r}_0 = f(\Delta h/h_v,\ Pr,\ B) \tag{15}$$

where

$$B = \rho_p^{1/3}\rho_g^{1/4}\alpha^{1/4}\mu^{1/2}/m_p^{11/12}\rho_s\dot{r}_o(1-W)$$
$$= k(\rho_p\rho_g\alpha/r_p)^{1/4}\ \mu^{1/2}/\rho_s\dot{r}_o(1-W) \tag{16}$$

Using "typical values" for $(\Delta h/h_v)$ and the Prandtl Number and substituting for gas density he arrives at, for a given formulation,

$$\dot{r}/\dot{r}_o = f_s[K_p(P_c\alpha)^{1/4}/m_p^{11/12}\dot{r}_o] \tag{17}$$

where K_p is a constant dependent only on the formulation. The function f_s calculated by Crowe is plotted in Fig. 4. Since it increases monotonically (and in fact nearly linearly) as shown in Fig. 4, the model predicts that $\dot{r}/\dot{r}_o$ decreases as the size of the agglomerate increases. Crowe then attributes the decrease in burning rate with time to this increase in agglomerate mass with time. However, since we still have no a priori knowledge of *how* the agglomerate mass increases with time, the model is still effectively qualitative rather than quantitative. In addition, the model predicts that the agumentation ratio increases with the ¼th power of pressure or acceleration magnitude and is inversely proportional to the base burning rate, *at constant agglomerate size.* However, since it does not predict how agglomerate size varies with pressure, acceleration magnitude, or base rate, the model does not permit prediction of the variation of augmentation-ratio-vs-time-behavior with these parameters.

Next, Crowe turns his attention to the "quasi-equilibrium" period where the burning rate has tapered off to an asymptotic value. Here he modifies the previous model to account for the fact that the globules "grow to the point that surface tension can no longer maintain near-spherical shapes and they gradually deform to the platelets and sheets characteristic of the quasi-equilibrium period," The modifications to the

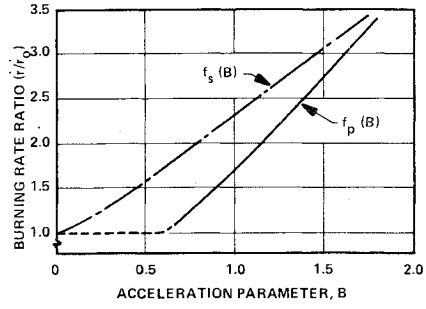

Fig. 4 Predicted variation of the burning rate ratio with the acceleration parameter.[12]

previously described model consist of allowing the globule to spread out to a point where its upper surface is flat and its thickness at any point may thus be determined from

$$\delta(\eta) = (P(\eta) - P_c)/\rho_P \alpha \tag{18}$$

where $P(\eta)$ is determined as before, and by changing the η_o criterion shown in Fig. 2 for the distorted sphere case to:

$$\left.\mathrm{d}\delta/\mathrm{d}r\right|_{\text{edge of globule}} = \tan\left[\cos^{-1}(\dot{r}_o/\dot{r})\right] \tag{19}$$

With these changes, he arrives at

$$\dot{r}/\dot{r}_o = f_p\left[K_p'(P_c\alpha)^{1/4}/m_p^{1/12}\dot{r}_o\right] \tag{20}$$

where f_p is plotted in Fig. 4 and K_p' is a constant dependent only on the propellant formulation. He then reasons that by this time the agglomerate size is so large that the rate of percentage change is sufficiently small that the change in $m_p^{1/12}$ with time is negligible, and thus the augmentation ratio is independent of time. If it is further assumed that this "quasi-steady" value of agglomerate size is not strongly dependent on the other independent variables, then the model may be used to predict the dependency of "quasi-steady augmentation ratio" on acceleration magnitude, base burning rate, and pressure. If, for example, over a range of acceleration level of 20 to 200g or pressure level of 100 to 1000 psia, the quasi-steady agglomerate size only varies by a factor of 2 to 4, then the effect of the change in agglomerate size on B will only be a factor of 1.06 to 1.12, compared to the direct effect of the change of P_c or α of a factor of 1.78—in this case, neglect of the dependency of agglomerate size on the independent variables does not seriously compromise the model predictions. With such an assumption, Eq.(22) reduces to

$$\left.\dot{r}/\dot{r}_o\right|_{\text{quasi-steady}} = f_p\left[K_p''(P_c\alpha)^{1/4}/\dot{r}_o\right] \tag{21}$$

Crowe was able to fit quite well the quasi-steady augmentation ratio data for a series of aluminized composite propellants with polybutadiene binder for 170 psi $< P_c <$ 900 psi, 30g $< \alpha <$ 650g, 0.16 ips $< \dot{r}_o <$ 1.03 ips with the expression

$$\begin{aligned}
&\dot{r}/\dot{r}_o = 1 \text{ for } (\alpha P_c)^{1/4}/\dot{r}_o < 50 (g\text{'s-psia}^{1/4}/\text{in/sec}) \\
&\dot{r}/\dot{r}_o = 1 + 0.0184[(\alpha P_c)^{1/4}/\dot{r}_o \\
&\quad -50] \text{ for } \alpha P_c^{1/4}/\dot{r}_o > 50
\end{aligned} \tag{22}$$

The final model of acceleration-induced burning rate augmentation of metalized propellants to be reviewed here was published by Ishii, Niioka, and Mitani[8] in 1973. These authors use much of the procedure of Crowe and Willoughby, but attempt to treat the agglomerate formation mechanism in order to calculate the agglomerate size as a function of the various independent parameters. However, they ignore all of the data indicating that the agglomerate size is time-dependent and assume that metal is consumed (and oxide product removed from the agglomerate) at a rate equal to the rate at which fresh aluminum is added. They then write a steady-state mass balance on the agglomerate as

$$\Pi \dot{r} p_s W D_{\text{pit}}^2 G_a/4 = \dot{r} p_s W N_s G_a = k r_p \tag{23}$$

where k is a constant related to the d^2 law burning rate constant k_b by $k = k_b \rho_p \Pi/2$. However, Ishii, et al., did not attempt to use aluminum burning rate data from the literature but instead treated their constant k as an empirical constant. By this procedure, with k being considerably larger than physically realistic from $k = \mathrm{k}_b \rho_p \Pi/2$, a steady-state value of agglomerate size can be calculated. Ishii, et al.,[8] calculates G_a in the same manner that Glick calculates G (Eq. 6) except that a (1-W) term is included in the calculation of gas velocity to correct for the fact that part of the propellant efflux is solid rather than gaseous.

The number of pits (or agglomerates) per unit area of combustion surface, N_s, is assumed to be proportional to the product of the number of retained aluminum particles per unit area of combustion surface at ignition and the average volume of those retained particles. They calculate the former quantity as the two-thirds power of the number of particles retained per unit volume: thus it is actually the number retained per unit area while the propellant regresses through one layer of particles.

The number of particles retained per unit volume of propellant and the mean volume of the retained particles are calculated straightforwardly with use of a drag force-body force balance to determine a critical particle size and assumption of a log-normal distribution of particle radii with known mass median and variance. The resulting equations are combined with Eq. (23) to yield an expression for the agglomerate size in terms of the augmentation ratio and various independent parameters such as pressure, base rate, and acceleration level: this expression is then substituted back into the two expressions relating the augmentation ratio and the width of the gap between the agglomerate and the pit base to finally permit calculation of the augmentation ratio, as a function of independent parameters alone, with the empirical constant being fit at an experimental point.

Nonmetalized Propellant Models

At this point, let us turn our attention to the two existing models for burning rate-acceleration interactions in nonmetalized propellants. Glick and co-workers[9,10] in 1966 extended Summerfield et al.'s[26] granular diffusion flame model for composite solid propellant combustion to include acceleration effects. Their model basically assumes that the propellant surface pyrolyzes to yield vapor pockets of gaseous fuel in a continuum of gaseous oxidizer. As the inhomogeneous mixture of gases flows away from the surface the fuel vapor pockets are assumed to burn in a diffusion-limited mode until completely consumed. Part of the heat released in this combustion process is transferred back to the surface by conduction to preheat and pyrolyze the solid. Since the heat is transferred by conduction with a temperature gradient driving force, the rate of heat feedback depends on the distance from the surface at which the heat is released, decreasing with increasing distance. As derived by Summerfield, the heat balance may be written as (approximating the gas phase temperature profile as being linear)

$$\dot{r} = \lambda_g (T_{\text{flame}} - T_{\text{surf}})/\rho_s \delta_r h_v \tag{24}$$

The quantity δ_r is the product of the velocity of the fuel vapor pocket away from the surface and the lifetime of the pocket (determined from diffusional considerations)

$$\delta_r = V_{fv} \tau_{fv} \tag{25}$$

Since the density of the fuel vapor is greater than that of the oxidizer, the fuel vapor pocket will move relative to the oxidizer vapor continuum flow in an acceleration field. Thus, an inward directed acceleration will cause the pocket to lag behind the oxidizer vapor in moving away from the surface, lowering V_{fv}, while an outward directed acceleration will increase V_{fv}. Either type of acceleration, by inducing relative motion and thus increasing the pocket Sherwood Number, will decrease τ_{fv}. Therefore, an inward directed acceleration should raise $\dot{r}$ while an outward directed acceleration may raise it, lower it, or leave it unchanged, depending on the

relative magnitudes of the two effects. Glick assumes that the relative velocity quickly reaches a steady-state value in which the acceleration body forces are just offset by the drag of the surroundings on the vapor pocket. Thus,

$$V_{fv} = U_{gs} - \left[\frac{\alpha d_{fv} \Delta\rho}{\rho_{v,ox} C_d} \right] \cos\phi \quad (26)$$

The fuel vapor pocket lifetime is calculated as

$$\tau_{fv} = d_{fv}^2 / D_g Sh \quad (27)$$

Equations (24-27) are combined by Glick to yield, after considerable manipulation

$$-\dot{r}/\dot{r}_o = k \frac{(Gr)^{1/2}\cos\phi}{C_d^{1/2}(Re_o)} + \left[k^2 \left[\frac{(Gr)\cos^2\phi}{C_d (Re_o)^2} \right] + \frac{Sh}{2} \right]^{1/2} \quad (28)$$

where k is a constant dependent only on propellant composition. With application of Stokes law and substitution of an empirical expression for the Sherwood Number, Eq.(28) is converted to

$$\dot{r}/\dot{r}_o = \frac{k_1 (Gr)\cos\phi}{(Re_o)} + \left[\left[\frac{k_1 (Gr)\cos\phi}{Re_o} \right]^2 + 0.28 (Gr)^{1/2} (Sc)^{0.35} + 1 \right]^{1/2} \quad (29)$$

where k_1 is treated as an empirical constant to be determined by data fitting.

It has been estimated by numerous authors that a heat feedback flux of at least 200 cal/cm^2 sec is required for combustion of typical composite solid propellant at 1 cm/sec linear burning velocity. The thermal conductivity of the gases leaving the propellant surface is typically approximately $2 \cdot 10^{-4}$ cal/cm sec K and the difference between flame temperature and surface temperature is typically 2500K. Substitution of these values into a heat balance similar to Eq. (24) yields a required flame offset distance of about 25μ. For the above model to make sense, the fuel vapor bubble diameter must be considerably smaller than this—at the very most 2 or 3μ. Using Stokes Law for the drag coefficient, we may rewrite Eq. (26) for $\phi = 0$ (inward-directed acceleration) as

$$V_{fv} = U_{gs} - \alpha d_{fv}^2 \Delta\rho / 24 \mu_g \quad (30)$$

Substituting values of 981,000 cm/sec^2 (1000g) for α, $2 \cdot 10^{-4}$ cm for d_{fv}, $3.0 \cdot 10^{-3}$ gm/cm^3 for $\Delta\rho$, and $6.5 \cdot 10^{-4}$ gm/cm sec for μ_g, we find that the relative gas velocity ($V_{fv} - U_{gs}$) is approximately 0. 01 cm/sec compared with a typical value of U_{gs} of 300 cm/sec. Thus, it may readily be seen that V_{fv} is essentially unaffected by even a 1000g acceleration field. Similarly, substitution of this value of relative velocity into the empirical expression for Sherwood Number shows the latter to also be essentially unchanged; thus τ_{fv} is also unaffected and $\dot{r}$ (which is inversely proportional to the product of V_{fv} and τ_{fv}) is unchanged by the 1000g acceleration field. Thus, the Glick model seems quite unrealistic.

A second model to explain burning rate-acceleration interactions in nonmetalized propellants was developed by Sturm and Reichenbach[13,14] in 1968. This model is based on a model developed by Fenn[27] for composite solid propellants burning in a standard 1g field. Fenn's model (known as the phalanx model) is based on the assumption of preferential burning at the interface of oxidizer and binder, leading to penetration of the "phalanx" flame around and behind exposed oxidizer crystals. This phalanx flame may actually progress completely around the oxidizer crystal, detaching it from the binder matrix, before the particle itself has completely burned. It is then possible that the gases leaving the pyrolyzing binder will carry the particle away from the surface, resulting in a local depletion of oxidizer and consequent lowered burning rate due to lessened heat feedback.

In his model, Sturm assumes that fine oxidizer particles undergo this separation from the surface before appreciable consumption of the particles has taken place and, in the absence of a g-field, are immediately swept away from the surface. As a result, all of the potential heat release from their combustion with pyrolysis gases then occurs far from the propellant surface rather than immediately adjacent to the propellant surface where this heat is needed for further binder pyrolysis. The coarse oxidizer cut (in multimodal grinds) is assumed to remain at the surface even in the absence of a g-field. As the magnitude of an inward-directed acceleration field is raised sufficiently high, fine oxidizer particles will also be retained at the surface until the particle has burned to a size sufficiently small that the viscous forces just exceed the body forces. Thus, as inward-directed acceleration level is increased, more heat release will occur at the propellant surface, and the burning rate will increase. It is assumed that the burning rate increase is directly proportional to the amount of energy that is transferred to the propellant surface from decomposition of the fine oxidizer particles (and subsequent reaction with fuel) at the surface. As pointed out by Glick, [15] this assumption is somewhat inconsistent with the basic phalanx flame concept: "According to the phalanx flame model, burning rate is controlled by reaction at the AP/Binder interface. The mechanism by which energy released by freed AP particles (this means they have been bypassed by the phalanx flame) increases the phalanx flame rate is obscure." While the retention of AP particles near the surface should speed the process of eating through binder to exposure the next interface, it is not clear that this should result in the overall rate increase being proportional to the amount of energy transferred from the AP decomposition and combustion near the surface. In addition, the question of proper additivity of heat fluxes once again arises.

Sturm and Reichenback[13,14] write a heat balance around the extra energy release at the surface resulting from retention of part of the fine oxidizer as

$$\rho_s (\dot{r} - \dot{r}_o) h_v = \rho_s \dot{r} W_{of} h_c f_H J \quad (31)$$

which may be rearranged to yield (with lumping of W_{of}, h_c and h_v into one parameter, η_H, depending only on the propellant composition):

$$\dot{r}/\dot{r}_o = (1 - \eta_H f_H J)^{-1} \quad (32)$$

Sturm and Reichenbach interpret Fenn's model in such a manner as to conclude that f_H will be a function of pressure alone, being proportional to the one-third power of pressure. J is calculated as a function of acceleration magnitude, pressure, etc. by use of a drag force vs body force balance on the separated oxidizer particles.

As with most of the models described in this review, the heat balance employed in this model (Eq. 31) is questionable in that it implies that the ordinary flame zone heat feedback is unchanged from the nonacceleration case to the nonzero acceleration case at constant pressure. The Fenn[27] or Summerfield[26] models imply that this feedback rate is proportional to $(P^{2/3}/\dot{r})$ and additionally that f_H is proportional to this same group. Apparently, Sturm and Reichenbach[13,14] have substituted $\dot{r} = kP^{1/3}$ (true for these models in the diffusion-limited region in the absence of acceleration, but not necessarily true in the presence of acceleration) to arrive at the conclusion that the ordinary flame zone heat feedback is dependent only on pressure and that f_H is proportional to $P^{1/3}$, independent of any other parameters. However, a more correct treatment begins with writing the heat balances with

and without acceleration as

$$(kP^{2/3}/\dot{r}) + \rho_s \dot{r}_o W_{of} h_c f_H J = \rho_s \dot{r} h_v \quad (33)$$

$$kP^{2/3}/\dot{r}_o = \rho_s \dot{r}_o h_v \quad (34)$$

where f_H is now a function of P and $\dot{r}$. These equations may be combined with elimination of k to yield the following expression

$$\dot{r}/\dot{r}_o = (1 - \eta_H f_H J)^{-1/2} \quad (35)$$

The major objection to the Sturm and Reichenbach model is that observations of burning nonmetalized composite propellants tend to refute the basic concept of preferential interfacial burning. Since burning AP particles are not observed leading a propellant surface in a null acceleration field, it is difficult to see how an acceleration field will raise the burning rate by causing them to remain there. Thus, no matter how well the model may fit data, it appears to be on extremely tenuous grounds as far as representing physically the effects of acceleration fields on nonmetalized composite propellant combustion.

Conclusions

At this time, it appears that qualitative understanding of the major mechanism by which acceleration fields affect the combustion of metalized solid propellants has been achieved. However, quantitative models permitting *ab* initio calculations of observed behavior (including the time-dependency of the buring rate-acceleration interaction seen with metalized propellants) have not been developed. In addition, the effects of binder type, pressure, metal content and particle size, oxidizer particle size, and base burning rate on the acceleration sensitivity of metalized solid propellants (particularly the transient aspects) have not been clearly defined, since their effects on the trapped agglomerate size have not been successfully modeled. The quasi-equilibrium part of Crowe's unified model does show considerable promise in permitting prediction of the burning rate augmentation vs acceleration characteristics of metalized propellants as a function of pressure, base burning rate, etc., following the transient behavior period, though it is still not totally clear that the variation of agglomerate size in this period with the various independent parameters has negligible effect on the augmentation. With regard to nonmetalized composite propellants, the existing models appear to be on very poor physical grounds.

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